Color naming, unique hues, and hue cancellation predicted from singularities in reflection properties

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Abstract

Psychophysical studies suggest that different colors have different perceptual status: red and blue for example are thought of as elementary sensations whereas yellowish green is not. The dominant account for such perceptual asymmetries attributes them to specificities of the neuronal representation of colors. Alternative accounts involve cultural or linguistic arguments. What these accounts have in common is the idea that there are no asymmetries in the physics of light and surfaces that could underlie the perceptual structure of colors, and this is why neuronal or cultural processes must be invoked as the essential underlying mechanisms that structure color perception. Here, we suggest a biological approach for surface reflection properties that takes into account only the information about light that is accessible to an organism given the photopigments it possesses, and we show that now asymmetries appear in the behavior of surfaces with respect to light. These asymmetries provide a classification of surface properties that turns out to be identical to the one observed in linguistic color categorization across numerous cultures, as pinned down by cross cultural studies. Further, we show that data from psychophysical studies about unique hues and hue cancellation are consistent with the viewpoint that stimuli reported by observers as special are those associated with this singularity-based categorization of surfaces under a standard illuminant. The approach predicts that unique blue and unique yellow should be aligned in chromatic space while unique red and unique green should not, a fact usually conjectured to result from nonlinearities in chromatic pathways.

Keywords: Color perception, Reflection properties, World color survey, Unique hues, Hue cancellation

Introduction

Particular hues of red, green, blue, and yellow are considered as "pure colors", often used in color-classification schemes (Hering, 1920/1964; Boynton & Olson, 1987; Chichilnisky & Wandell, 1999). There are strong indications that the choice of these colors is not a mere cultural convention since these same colors are consistently singled out and given names even in widely separated human societies (Berlin & Kay, 1969; Regier et al., 2005). Furthermore, psychophysical tests provide measures of the fact that these colors appear unmixed, or "unique", in the sense that they seem to observers not to be tinted with any other color (Jameson & Hurvich, 1955; Valberg, 2001).

These facts, which are considered to be at the heart of color vision, have up till now not been adequately explained.

First, there is no known *physical* reason why certain colors should be perceived as unmixed or special. In particular, lights that elicit sensations judged to be unmixed by observers can actually be obtained by superposing other lights, and paints judged to be unmixed can be obtained by mixing other paints.

Second, no accurate *biological* basis for unique hues has so far been found. The trichromatic theory of human vision, based on three types of photoreceptors, would seem unable to explain the existence of exactly four unique hues. The opponent-process theory argues that there are four perceptually unique hues because of the existence of so-called red/green and blue/yellow neural pathways, but specialists in color vision agree that when the theory is confronted with available physiological data, it does not adequately predict which hues are observed to be perceptually unique (Knoblauch et al., 1985; Webster et al., 2000; Valberg, 2001; Kuehni, 2004).

Here we show theoretically and numerically that, contrary to what is usually thought, the existence of four special surface colors can be expected from trichromatic theory alone, when constraints satisfied by natural illuminants and surfaces are taken into account. The colors predicted to be special from this approach are precisely those which are found in empirical surveys to be most often given a name by different cultures throughout the world. We shall additionally see that the approach provides correct quantitative predictions for foundational facts of color science, namely psychophysical data about unique hues and hue cancellation.

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Terminology: accessible information

In this paper, we will call "accessible information" about a light the restricted information about the spectral composition of that light which is accessible through an organism's photopigment set: For humans, we can take for instance the three numbers corresponding to the photon absorption counts for each of the photopigment types present in human photoreceptors. We use the term "accessible information" rather than "spectral coding" or "photoreceptor excitations" because the two latter terms leave an ambiguity as to whether they refer to the number of photons absorbed by photopigments, or the neural coding of these numbers. We want to be clear that the constraint we analyze herein really is about photon catches and not neural code for color information: in particular, properties of linearity that we consider refer to catches, not to photoreceptor outputs.

A biological approach for reflection properties

The approach we propose involves constructing a biological restriction of the physicist's notion of reflectance. From the physicist's viewpoint, reflectance is the relationship that holds between the spectrum of the light illuminating a surface and the spectrum of the light reflected by that surface. To take into account the loss of information induced by photopigments, it is therefore natural for the biologist to study the relationship between the accessible information (instead of the spectrum) about the illuminant and the accessible information (instead of the spectrum) about the reflected light. We will show that, with respect to standard sets of natural illuminants, physical surfaces' reflectance induce a linear constraint between the accessible information about the illuminant and the accessible information about the light reflected by that surface. This suggests a biological conception of surface reflection properties as *linear operators* in the three-dimensional space of the accessible information.

The work of the paper will consist in studying the characteristics of these linear operators for a large number of natural and artificial surfaces. We shall show that for certain surfaces the operator is *singular*: instead of variations in the incoming light causing variations of the reflected light along all *three* directions of the accessible information space as is usually the case, in the singular cases, variations in incoming light only produce strong variations in the reflected light along one or along two directions. These singular cases where a surface affects incoming light in a simpler way than usual will be seen to predict, with surprising accuracy, data on color naming, unique hues, and hue cancellation.

Databases and numerical methods

Database for photopigments

We used the 10-deg Stiles and Burch Color-Matching Functions (CMFs), arguably the most reliable experimental data about the *in situ* sensitivity of the set of human photopigments (Stiles & Burch, 1959). Since our approach does not rely on a linear basis for the accessible information, we do not actually need to go back to the absorption of individual photopigments: we could simply perform our numerical analysis directly from CMFs. We checked that using Stockman and Sharpe (2000) cone fundamentals does indeed not bring any noticeable modification of the results. In the theoretical derivations below, we refer to photopigment absorptions since these are the original physical constraints.

Database for illuminants

We used a set of illuminants composed from 99 daylight spectra in Granada from Romero et al. (1997), 238 daylight spectra in forest from Chiao et al. (2000), and a Gaussian sample of 200 spectra constructed from basis functions S0, S1, and S2 derived by Judd et al. (1964). The rationale was to consider diverse sources of illuminants so as to avoid possible statistical artifacts resulting from considering a too specific ecological niche.

Database for reflectances

We used two sets of reflectances: one made from artificial surfaces, and one made from natural surfaces. The set of artificial surfaces was constituted by Munsell chips, whose reflectances were taken from the University of Joensuu (Parkkinen et al., 1989—http://spectral.joensuu.fi/). We chose the set of 1600 Munsell glossy chip reflectances because it is the one from which chips used for the World Color Survey were taken. The set of natural surfaces comprised 404 reflectances of leaves, petals, and barks from the grounds of Keele University in Staffordshire (Westland, Shaw, & Owens, 2000), 1862 reflectances of fruits and foliages from French Guiana and Uganda, made available by the University of Cambridge, 1413 reflectances of flowers and 246 reflectances of leaves from the Chittka dataset (Chittka & Menzel, 1992).

Numerical methods

All numerical computations were performed using Matlab 7 (Mathworks, Natick, MA). All spectra were resampled for compatibility at 5-nm intervals within the 400–700 nm band of the spectrum. Numerical estimation of integrals was performed by a simple dot product between sampled spectra. Standard algebraic computations such as multivariate regression and decomposition in eigenvalues and eigenvectors were performed using standard Matlab routines without any particular options.

Mathematical treatment

In this section, we first verify numerically that over our datasets surface reflectances can accurately be described by linear operators on accessible information. We then introduce the concept of noninteracting basis for accessible information and show how it can be used to define a singularity index for each surface in the databases. The Results section confronts the measured singularity for each surface to existing psychophysical data on color naming, unique hues, and hue cancellation.

Computing biological reflectance mapping

Consider an illuminated surface. Let $E(\lambda)$ denote the spectral power distribution of the light incident on the surface at each wavelength λ. Let $R_1(\lambda)$, $R_2(\lambda)$, and $R_3(\lambda)$ denote the absorption rate at each wavelength λ by photopigments present in the L, M, and S human photoreceptors. Then the accessible information about the incident illuminant can be described by the triplet $u(E) = (u_1(E), u_2(E), u_3(E))$, where

$$u_i(E) = \int R_i(\lambda)E(\lambda)\,d\lambda, \quad i = 1\ldots3. \tag{1}$$

Let $S(\lambda)$ denote the reflectance function of the surface, then the accessible information about the reflected light is the triplet $v^S(E) =$
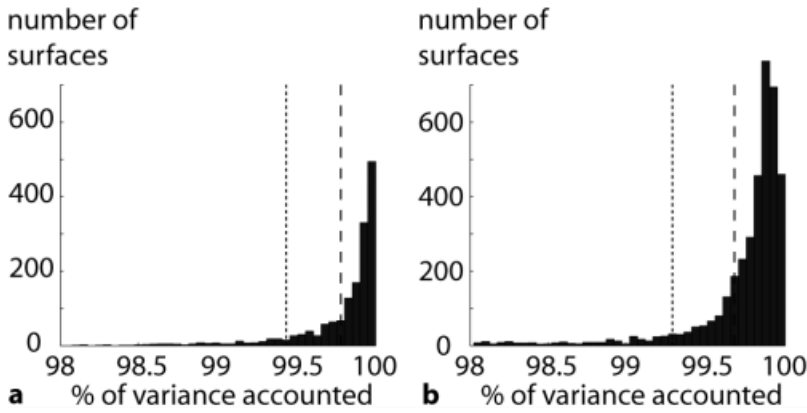
Fig. 1. Variance residuals for the set of Munsell chips (left) and natural surfaces (right). Plots show the number of surfaces as a function of the variance of the accessible information for the reflected light accounted for by a linear transformation of the accessible information for the incident light. Dashed line represents the mean value of the variance accounted for over the dataset, dotted line represents the value of the variance at least accounted for by 90% of surfaces.

$(v_1^S(E), v_1^S(E), v_1^S(E))$ (superscript S specifies that this information depends on the surface S considered, in addition to the illuminant E) where

$$v_i^S(E) = \int R_i(\lambda) S(\lambda) E(\lambda)\, d\lambda, \quad i = 1\ldots 3. \tag{2}$$

Now, instead of a unique illuminant assume a probability distribution over illuminants. Then, for a given surface S, the existence of a linear relationship between three-dimensional variables u and v^S, that is, the existence of a linear operator A^S in the accessible information space such that $v^S = A^s \cdot u$, can be assessed by computing the empirical variance of the residual $\eta = v^S - \hat{A}^S \cdot u$, where $\hat{A}^S$ is the 3×3 matrix obtained by a linear regression of v^S onto u computed from a sample of illuminants. For each surface of our dataset, we therefore performed a linear regression from a series of calculations simulating the illumination of the surface successively by each illuminant of our database of daylights.

Fig. 1 summarizes the variance of residuals obtained. Results show that the variance left after taking into account a linear relationship between u and v^S is negligible: in terms of average value over our two sets of surfaces, this residual represents 0.3% of the original variance of v^S for the set of natural surfaces and 0.2% for the set of Munsell chips, and in terms of quantiles, more than 90% of both natural and artificial surfaces have a residual smaller than 0.7% of the original variance of v^S. This confirms that for the purpose of color vision, we can restrict our view of physical reflection properties to a biological conception of linear operators in the three-dimensional space of the accessible information.

This result was expected from the well-known low dimensionality of the set of natural illuminant spectra. The 400–700 nm band of daylight spectra are well approximated by weighted sums of only a few functions, presumably as few as three (Judd et al., 1964; Maloney, 1999; Chiao et al., 2000). In these conditions, let $E_1(\lambda)$, $E_2(\lambda)$, $E_3(\lambda)$ be such basis functions for illuminant spectra and ε_1, ε_2, ε_3 be the coordinates of $E(\lambda)$ in this basis so that

$$E(\lambda) = \sum_{k=1}^{3} \varepsilon_k E_k(\lambda). \tag{3}$$

Build 3×3 matrices $U = [u(E_1), u(E_2), u(E_3)]$ and $V^S = [v^S(E_1), v^S(E_2), v^S(E_3)]$, then we have $v^S(E) = V^S(\varepsilon_1, \varepsilon_2, \varepsilon_3)^T$ and $u(E) = U(\varepsilon_1, \varepsilon_2, \varepsilon_3)^T$, therefore if U is invertible (basis functions yield linearly independent photon catches triplets), we find that $v^S(E) = V^S U^{-1} u(E)$. Thus, the three dimensionality

of the illuminant space implies a linear relationship between u and v^S.

Note that the linear operator $A^S = V^S U^{-1}$ does not depend on a probability distribution of illuminants within the linear model of eqn. (3), but recall also that the illuminant space spanned by such a model is itself determined by a probability distribution over the set of all possible illuminants. Note finally that even though it provides an intuitive rationale, the three dimensionality of the illuminant space (questioned for instance by the work of Romero et al. (1997)), is not mandatory to find a linear relationship between u and v^S since some aspects of the spectrum to be modeled by basis functions might have little influence on both u and v^S.

Noninteracting bases for accessible information

Since the biological analog A^S of a physical reflectance $S(\lambda)$ is a simple linear operator in the accessible information space, we can express it in terms of eigenvalues and eigenvectors, the possibility of such a decomposition in the complex field relying on a rather weak assumption (see Appendix on real and complex eigenvalues). If the decomposition exceptionally turns out to occur in the real field, then eigenvalues and eigenvectors have straightforward interpretation. Indeed, the three eigenvectors of A^S will provide a basis for the accessible information space such that elements of this basis do not mutually interact with each other when they are reflected by the surface $S(\lambda)$, and are simply individually scaled by the associated eigenvalues.

It is very simple in this basis to determine, for the surface S, the accessible information about the reflected light from the accessible information about the illuminant: let $\{r_1^S, r_2^S, r_3^S\}$ and $\{C_1^S, C_2^S, C_3^S\}$ be the three eigenvalues and three eigenvectors of the operator A^S, and assume that $u(E)$ has coordinates $(\alpha_1^S, \alpha_2^S, \alpha_3^S)$ in the basis* $\{C_1^S, C_2^S, C_3^S\}$, then

$$u(E) = \sum_{j=1}^{3} \alpha_j^S C_j^S \Rightarrow v^S(E) = A^S u(E)$$

$$= \sum_{j=1}^{3} \alpha_j^S A^S C_j^S$$

$$= \sum_{j=1}^{3} r_j^S \alpha_j^S C_j^S \tag{4}$$

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*Note that although $u(E)$ is not dependent on the surface considered, the set of eigenvectors of A^S is, and therefore the coordinates of $u(E)$ in that basis do depend on the surface.

that is, $v^S(E)$ has coordinates $(r_1^S \cdot \alpha_1^S, r_2^S \cdot \alpha_2^S, r_3^S \cdot \alpha_3^S)$ in this same basis. This also means that it is easy in this basis to discount the illuminant from the accessible information about the reflected light: it suffices to divide independently each coordinate of the accessible information about the reflected light by the same coordinate of the accessible information about the illuminant. Note that this is only possible in this basis, and therefore *a fortiori* only possible if A^S has real eigenvalues.

It is a remarkable fact that noninteracting bases can be found for most physical surfaces: in fact the majority of linear operators associated with reflectance functions of our dataset turn out to be decomposable in the real field. 88% of surfaces over our artificial and natural datasets have real eigenvalues, and for the remaining surfaces the bound $\rho \sin(|\theta|/2)$ (see Appendix on real and complex eigenvalues) never exceeds 0.023 for the set of Munsell chips and 0.012 for natural surfaces. Therefore, it is possible for each surface to find a noninteracting, or approximately noninteracting, basis $\{C_1^S, C_2^S, C_3^S\}$ and real reflection coefficients $\{r_1^S, r_2^S, r_3^S\}$. The point is that these bases are different from surface to surface. Fig. 2 shows examples of light changes associated with such bases, for two different surfaces.

The three reflection coefficients defined in this way for each surface then almost provide a biological analog of the physicist's reflectance function. Indeed, just as the physicist describes the reflectance function in terms of how strongly a surface reflects each of an infinity of noninteracting monochromatic light components, we here can describe a biological analog in terms of the three numbers which define how strongly the surface reflects each of the three noninteracting components of the accessible information. It must be noted that coefficients $\{r_1^S, r_2^S, r_3^S\}$ do not depend for instance on the fact that we are using color-matching functions rather than cone absorption curves to *represent* the accessible information. Indeed, CMFs are essentially a linear combination of cone absorption curves (Wandell, 1997), and therefore using them rather than cone absorption curves merely leads to a linear change of coordinates for the accessible information. Since we did not use any *a priori* particular coordinate system to define noninteracting basis and reflection coefficients, this has no influence on reflection coefficients.

Whereas reflection coefficients are an analog of the physicist's reflectance function, the analogy is not complete because in contrast with the physical basis of monochromatic spectra and as already stressed, the biological basis may vary from surface to surface (hence the superscript S in C_i^S). Yet the existence of a unique basis actually is an implicit assumption of several standard approaches to color constancy. The accessible information for the illuminant being the accessible information for the light reflected by a white surface, the existence of a unique basis for a set of surfaces is, in addition to the gray world hypothesis, the condition for a von Kries algorithm in this basis to achieve color constancy of a scene constituted by such surfaces. From this viewpoint, our work relates to previous work on spectral sharpening by Finlayson et al. (1994). As for cone ratios between surfaces in a scene (Nascimento et al., 2002), their rigorous invariance can be seen to require a unique, and additionally very specific, basis for these surfaces: the basis defined by cone catches. In this paper, however, we will not elaborate on the existence of a unique basis for a set of surfaces and stick to the investigation of the biological constraints induced by each surface *individually*.

Singular reflection properties

Contrary to the situation in physics where there are an infinity of reflection coefficients, the biological case, with only three coefficients, presents asymmetries determined by a few specific configurations in the magnitudes of these coefficients. A first such singular case is the case when all three reflection coefficients are about equal, that is, when the three noninteracting components of incoming light are equally transmitted. This case clearly corresponds to achromatic surfaces, since the spectral composition (or rather, what is biologically accessible about this composition) remains unchanged by the surface. Examples of such cases will be white surfaces and black surfaces, with all three reflection coefficients equal to 1 or to 0, respectively.

Another type of singularity will consist in cases when variations in incoming light only produce strong variations in the reflected light along one or along two directions. Like black and white, we also expect that these cases will have a special perceptual status. To quantify the extent to which a surface approaches one of these two special cases, we considered the two ratios $\beta_1 = r_1/r_2$ and $\beta_2 = r_2/r_3$, where (r_1, r_2, r_3) are the reflection coefficients sorted in decreasing order. Maxima of β_1 correspond to surfaces that reflect one of the elements of the noninteracting basis much more significantly than the two others; maxima of β_2 correspond to surfaces that reflect two of elements much more significantly than the third. Because we would like for simplicity to have a unique index summarizing these two cases, we could consider just taking the maximum of β_1 and β_2. But it turns out that for our dataset of surfaces, β_1 peaks at about 7 while β_2 peaks at about 21, hence such an index would essentially pinpoint how close we are from the second case. To deal with this issue, we first normalized each of β_1 and β_2 to their maximum value so that value 1 means for both "maximum singularity", and only then gathered the two ratios into a single value by taking their maximum. To sum up, we defined our singularity index as $\beta = max(\beta_1/\beta_1^{max}, \beta_2/\beta_2^{max})$, where β_1^{max} and β_2^{max} are the maximum values of each β value over the dataset of surfaces.
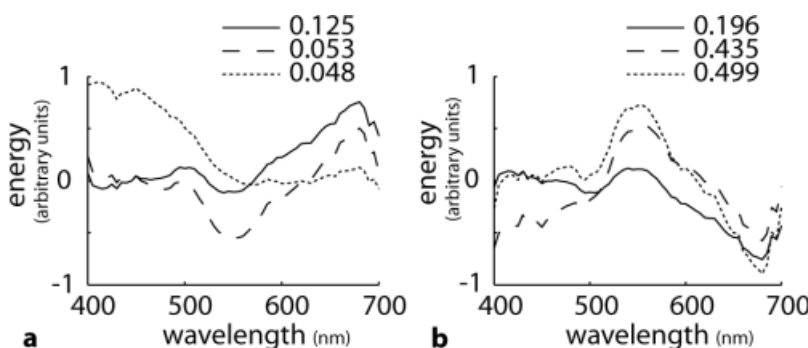

Fig. 2. Illuminant changes associated with a noninteracting basis of the accessible information: (a) Munsell chip 10 R 2/6, (b) Munsell chip 5 BG 7/8. The accessible information for these components of the illuminant is merely scaled by the surface, and by the amount specified in the legend.

Results

The Mathematical treatment section has shown that there is for each surface a linear operator which describes the way the surface transforms accessible information about illuminating light to accessible information about reflected light. We have also shown how to define a *singularity index* which quantifies the degree to which this linear operator corresponds to the special cases where the variation of reflected light, under changes in illuminant, is confined essentially to only one or two component directions, rather than being distributed over three component directions, as is normally the case. We now confront the singularity index, computed over large databases of surfaces with empirical data on color naming, unique hues and hue cancellation. We stress that the singularity index was calculated from existing databases of illuminants, photoreceptor fundamentals, and illuminants, without parameter adjustment of any kind.

Singular reflection properties and the World Color Survey

Fig. 3b plots the value of the singularity index for a standard set of surface patches, namely the colored chips used to study color categorization in cross-cultural studies (the World Color Survey, Berlin & Kay, 1969, made available by the International Computer Science Institute, Berkeley). The chips most often given a name by widely separated human cultures (Fig. 3a), and which we call "red", "yellow", "green" and "blue" in English, can be seen to be within one chip of those having maximally singular reflecting properties. According to the index we have defined, these cases are reached for chips labeled G2, C9, F16, and H31, with reflection coefficients {0.56,0.08,0.08}, {0.89,0.66,0.03}, {0.29,0.07,0.04}, and {0.27,0.07,0.07}.

Two striking facts should be noted from Fig. 3. First, the figure shows that there is a biological basis for the fact that precisely four categories of colored surfaces should be perceived as distinctive: these are singular in the sense that instead of reflecting three components, they reflect essentially only one or two components of the light described by biologically accessible information. The fact that a finite number of categories for reflection properties emerge from an infinity of possible reflectances, and further that four categories emerge from taking into account a trichromatic constraint on the visual system, already is a notable result from this biological reading of reflection properties. A second notable result from Fig. 3 is that these four categories of surfaces precisely
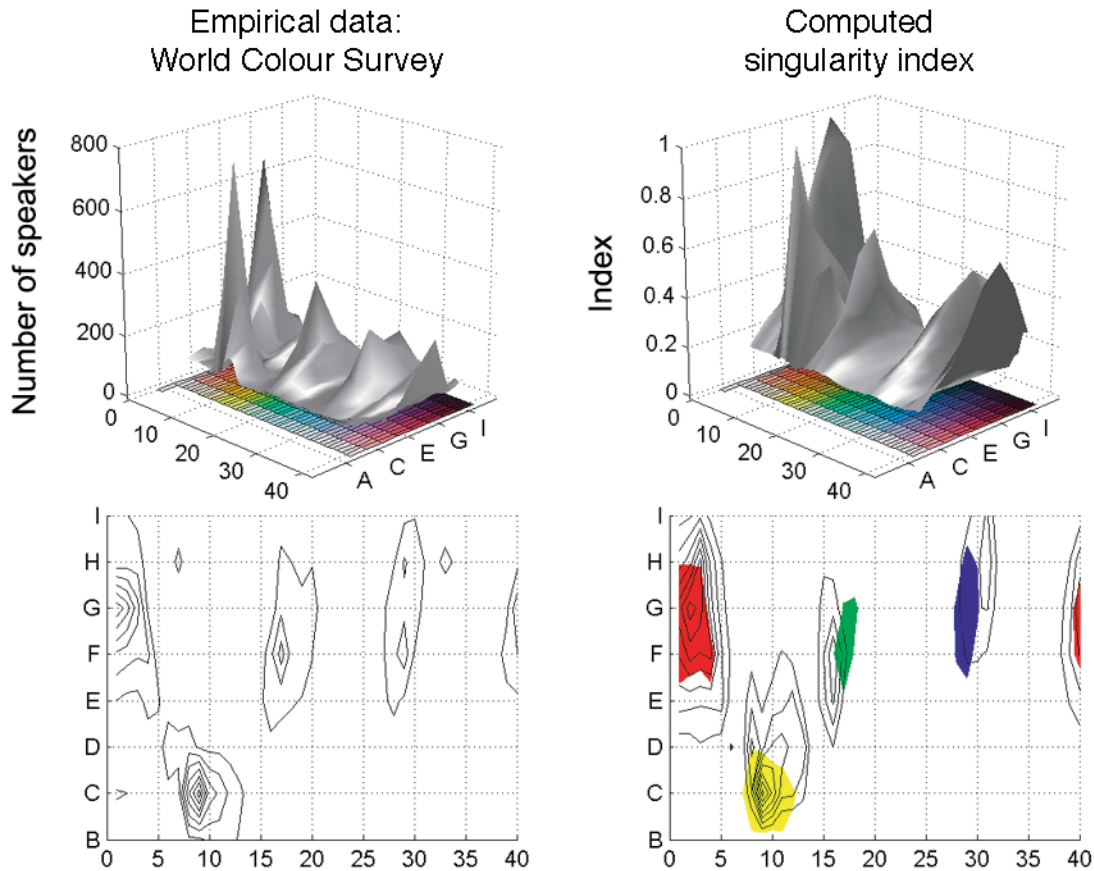

Fig. 3. Color terms across languages, and surface reflecting properties. Left panels show the number of speakers in Berlin and Kay's World Color Survey of 110 languages having a term that designates each of a set of colored surfaces. There are four main maxima, corresponding to four universal "focal colors": red, yellow, green, and blue. Right panels show our theory-based calculation for the maximum of the two ratios defined by the first *vs.* second, and second *vs.* third reflection coefficients (see text) for these surfaces. For comparison with the empirical data, colored areas in the bottom right panel correspond to those chips in the World Color Survey that were named by more than 20% of the maximum number of speakers. Letter and number notations for chips are those used in the World Color Survey.

correspond to colors most often given a name across widely separated cultures, namely red, yellow, green, and blue.

Both these facts, not previously noticed, have been obtained without appealing to neural mechanisms that underlie opponent channels in the visual system. They are merely a consequence of the asymmetries in surface reflecting properties induced by the limited way biological photoreceptors sample physical spectra. It could thus be argued that the reason the colors "red", "yellow", "green", and "blue" are so often singled out among all other colors as being worth giving a name, is that surfaces of these colors have the particularity that they alter incoming light in a simpler way than other surfaces: red, green, and blue surfaces reflect essentially one component. Yellow surfaces are also simple as compared to other surfaces in that they reflect two components.

Singular reflection properties and unique hues

The coincidence between singularities in reflecting properties and cross-cultural linguistic color categories suggests that beyond accounting for anthropological results, singularities in reflecting properties might also be correlated with more precise data collected by psychophysicists about color perception. To investigate this, we need to link our results about colored surfaces under natural illuminants to psychophysical results where observers classically face "aperture colors". Such stimuli are generated, roughly speaking, by sending lights of controlled spectral composition directly into the eye rather than by natural viewing of a colored surface. To compare our approach with the results of these experiments, we made the minimal conjecture that the nervous system interpreted such stimuli as being the result of reflection from a colored surface of light deriving from the most common illuminant type, known as D65. This provides a simple way to transpose the index concerning surface properties to an index concerning photon capture by human photopigments.

To simplify the representation of this transposed index, we reasoned that although it is known that natural reflectances are not accurately described by using sums of only three basis functions, such models nonetheless represent a first approximation that suggests that the index should be roughly invariant by scalar multi-

plication of photon catches. Indeed, if three-dimensional models of reflectance were exact, then the transposed indexes for proportional photon catches would be the indexes of proportional reflectance functions, and hence would be equal since we consider *ratio* of eigenvalues. As a result, the transposed index is expected to vary essentially on a projective structure, and we can plot the index using (x, y) CIE 1931 chromatic coordinates (Wandell, 1997).

Fig. 4a shows the transposed index based on the set of Munsell chips. For natural surfaces on the other hand, plotted in Fig. 4c, we found (as noted elsewhere, see for instance Westland et al., 2000) that the gamut spanned in chromaticity space by natural surfaces was smaller than that spanned by Munsell chips, and in particular too small in the green area to get a readable plot. We therefore artificially saturated the reflectances by generating a reflectance function $\tilde{S}(\lambda) = S(\lambda) - \min S$ for each natural reflectance of our database. While this manipulation must be kept in mind when reading Fig. 4c, it seems unlikely that the correspondence found then with Fig. 4a is a result of this mere saturation.

The crests of the index in these figures pinpoint stimuli associated with the most singular colored surfaces. It is a remarkable fact that these loci can be fitted by four half-planes in the full space of the accessible information that share as an edge the central axis defined by achromatic surfaces under the hypothesized illuminant, in this case D65 (Fig. 4b). These half-planes determine specific wavelengths at their intersections with the monochromatic locus, and therefore identify the wavelengths producing stimuli that are most exemplary of singular surface reflecting properties. This provides a prediction for the monochromatic lights that we might expect subjects will report as being "unique hues" in psychophysical experiments. As seen in the figure, the predictions are very close to observed empirical data on unique hues. Our approach is thus able, without any parameter adjustments, to predict psychophysically measured unique hues purely from known biological and physical data (photopigment absorption curves, and illuminant spectra and reflectance functions, respectively).

While the hypothesized illuminant, D65 in Fig. 4, could be suspected to play an essential role in the predictions of unique hue wavelength in this approach, this conjecture turns out to be wrong, at least for the case of red, green, and blue: this is because these
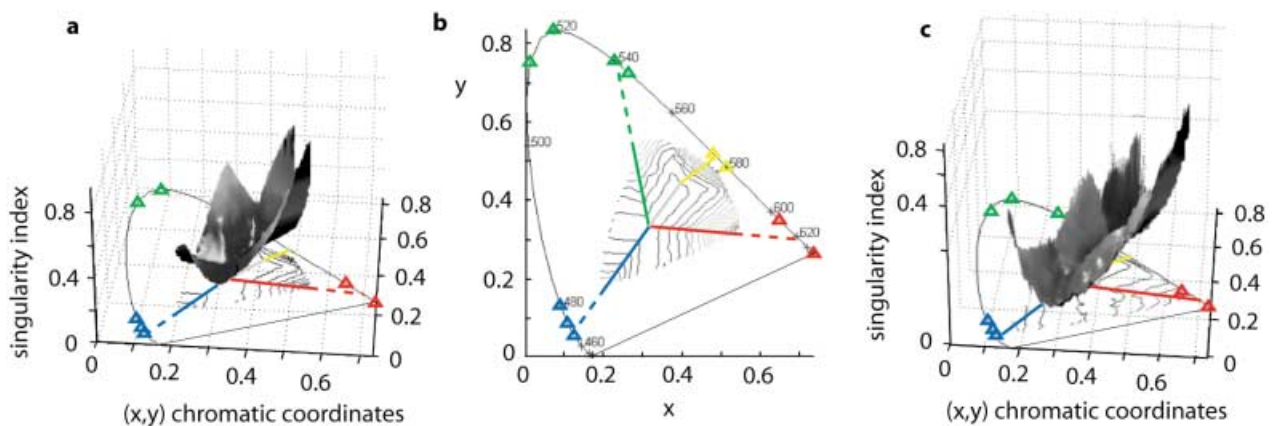

Fig. 4. Surface reflecting properties and unique hues. (a) Singularity index computed from the set of Munsell chips under daylight D65, plotted as a function CIE chromatic coordinates. Crests of the index are projected onto the ground plane as solid colored lines, marking the loci of maximum singularity. (b) Crests of the index extended out to the monochromatic locus. Colored triangles: psychophysical measures of unique hues from several sources given in Table 1. (c) Singularity index computed from a set of natural surfaces under daylight D65, plotted as a function CIE 1931 chromatic coordinates.

surfaces reflect only one component of the accessible information about the incident light and therefore the accessible information about the reflected light varies little in chromaticity as the hypothesized illuminant changes. It turns out numerically that the wavelength of unique yellow is also not strongly dependent either on the temperature of the illuminant: as the illuminant hypothesized to interpret tristimuli varies from temperature 3000°K to 20,000 °K, the wavelength of the unique yellow determined by our approach varies from 580 nm to 570 nm.

Another issue concerns the variability of observed unique hues. While there is evidence that visual experience plays an important role in the perception of unique hues as compared to phylogenetic neuronal specifics (Brainard et al., 2000; Lotto & Purves, 2002; Neitz et al., 2002; Sugita, 2004), it has until now been unclear what features of visual experience could influence, or arguably determine, unique hues. A rough indication about the expected variability of a unique hue within our approach, under the assumption that the singularity is relevant if it is above some threshold, can be given by the span over which the singularity index is "large", for instance greater than half of its value for the unique hue. Table 1 shows a comparison of these predictions with empirically observed unique hues and demonstrates that the specific pattern of variability found between observers (Kuehni, 2004) for the different unique hues is in agreement with what is expected from our approach. This is further support that singularities in the biologically accessible surface reflectance properties may indeed constitute the relevant features for understanding unique hues.

A last point concerns the well-established fact that unique yellow and blue are aligned in chromaticity space, while unique red and green are not. This is usually considered to result from a quirk of the neural system's structure (Chichilnisky & Wandell, 1999; Valberg, 2001; Knoblauch & Shevell, 2001), while our work suggests that it could rather reflect a physical fact about the information processed by the visual system: as seen from Fig. 4, the crest in the singularity index for red and green are almost colinear, whereas this is clearly not the case for blue and yellow.

Singular reflection properties and hue cancellation

A further consequence of the findings presented in this paper relates to the well-known phenomenon of hue cancellation. Hue cancellation quantifies the fact that the addition to a light that appears bluish of a certain amount of light that appears yellowish produces a light that appears neither bluish nor yellowish, and the same for lights that appear reddish or greenish. This has been considered as evidence that color perception was mediated by two sensation continua, redness–greenness and yellowness–blueness (Jameson & Hurvich, 1955; Valberg, 2001). Yet it is a fact, noted for instance by Mollon and Jordan, 1997, that hue-cancellation data are a direct consequence of the loci of unique hues: in Fig. 5a, the dash–dot cancellation curve were derived from computing the intensity of a monochromatic yellow light to add to a bluish light so that the corresponding stimulus is on the locus defining a unique hue different from yellow or blue (or the achromatic locus), and similarly for the dashed curve with red and green lights.

Since we have an account of the loci of unique hues from singular surface reflection properties, we can make predictions about hue cancellation without appealing to equilibration of "sensation channels" (Fig. 5b). Thus, we argue that hue-cancellation data cannot be considered as conclusive evidence for a Müllerian theory of two sensation continua. Obviously, this does not deny the importance of color-opponent neuronal pathways for color vision: in particular, they optimize information transfer from the retina (Buchsbaum & Gottschalk, 1984; Ruderman et al., 1998), and explain aftereffects (McCollough, 1965). But it casts doubt on the claim that this neuronal specificity on its own straightforwardly determines a fact about our conscious perceptual experience: the existence of a few special hues.

Conclusion

We have shown how by considering the information accessible *via* human photoreceptors about incoming light, it is possible to

Table 1. *Comparison between psychophysical measurements of unique hues (Kuehni, 2004) and predictions based on the structure of the singularities of surface reflecting properties. EOS means End Of Spectrum.*

Data set	Subjects	Unique yellow		Unique green	
		Mean (nm)	Range (nm)	Mean (nm)	Range (nm)
Schefrin	50	577	568–589	509	488–536
Jordan & Mollon	97	—	—	512	487–557
Volbrecht	100	—	—	522	498–555
Webster (a)	51	576	572–580	544	491–565
Webster (b)	175	580	575–583	540	497–566
Webster (c)	105	576	571–581	539	493–567
Prediction	—	575	570–580	540	510–560

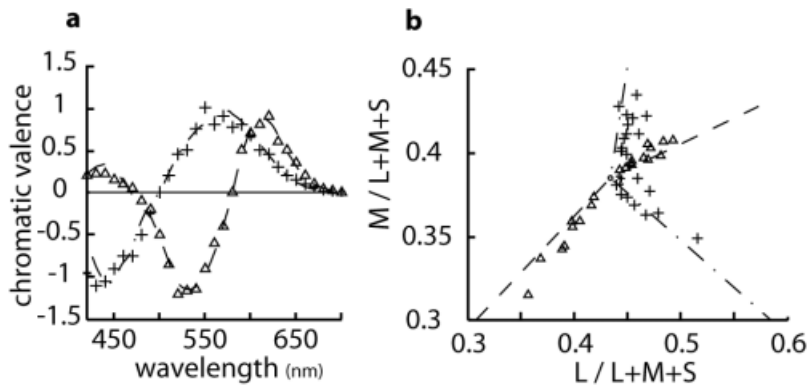| Data set	Subjects	Unique blue		Unique red	
		Mean (nm)	Range (nm)	Mean (nm)	Range (nm)
Schefrin	50	480	465–495	—	—
Jordan & Mollon	97	—	—	—	—
Volbrecht	100	—	—	—	—
Webster (a)	51	477	467–485	EOS	—
Webster (b)	175	479	474–485	605	596–700
Webster (c)	105	472	431–486	EOS	—
Prediction	—	465	450–480	625	590–EOS+

Fig. 5. Prediction of hue cancellation and equilibrium loci. (a) Classic empirical data from Jameson and Hurvich, 1955: Crosses show the relative intensity of yellow (negative ordinate) or blue (positive ordinate) lights added to monochromatic lights of different wavelength so that the observer judged the stimulus as neither yellow or blue. Triangles present similar data for red and green lights. Curves: predictions based on the observer's monochromatic unique hues. (b) Crosses show yellow–blue equilibrium, triangles show red-green equilibrium for one observer, from Chichinisky and Wandell, 1999. Dashed lines: loci for unique hues predicted from singular reflection properties.

characterize reflecting surfaces by a *singularity index*. This index quantifies the degree to which the surface reflects light in a simpler way than usual. Whereas usually a surface will cause variations in accessible information about reflected light along all *three* directions of the accessible information space, singular surfaces only produce strong variations along one or two directions.

We showed how the singularity index, as calculated over a large database of surfaces and illuminants, correctly predicts which surfaces will be given names across widely separated cultures. We then showed how the singularity index can be applied to lights instead of surfaces in order to predict empirically observed unique hues. Data on hue cancellation is therefore also well explained by this approach.

In addition to the surprisingly accurate predictions of classic psychophysical data, made without any parameter adjustments, the approach proposed here has interesting philosophical implications. Colors are a genuine quandary to science because they forcibly instantiate the difficulty of explaining the quality of perceptual awareness. While the dominant view today is that perceptual qualities are determined by specificities of cortical representations (Valberg, 2001; Crick & Koch, 2003; Zeki, 2003), a different stance about the role of nervous activity can be taken. Rather than conjecturing that nervous activity determines the quality of sensations, a conjecture that opens up an "explanatory gap" (Levine, 1983; Chalmers, 1996), we propose instead to exploit the less controversial fact that this neural activity enables the sensorimotor involvement of the organism and its cognitive abilities. What determines the perceived quality of sensations, we suggest then, are intrinsic features of the overall constraints imposed on the interaction of the organism with its environment (O'Regan & Noë, 2001). Colors have for long been used as an exemplary counter-argument to such functional accounts of sensations, yet the common rationale and surprisingly accurate quantitative account provided here for color naming, unique hues and hue cancellation appears to lend credence to this approach.

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Appendix: Real and complex eigenvalues

We recall that the eigenvalues of a linear operator A on a vector space V are the roots (possibly complex) of the characteristic polynomial $P(x) = \det(A - x \cdot Id)$. In the case where all roots have multiplicity 1, then there is a basis of V made of corresponding eigenvectors, that is, non-null vectors u_i such that $Au_i = x_i \cdot u_i$ for some complex values x_i. Only a set of operators with null measure do not have roots with multiplicity 1: hence, with probability 1, A can be decomposed in terms of complex eigenvalues and eigenvectors. In the case where V is three dimensional and A is real, then A has one real root and the two other roots are either real, or conjugate: $\rho e^{i\theta}$ and $\rho e^{-i\theta}$. Recalling that from a norm $\|\cdot\|$ on V, it is possible to define a norm on linear operators by $|\|A\|| = \max\{\|Au\|, \|u\| = 1\}$, it can be shown that for all possible norms on V, the distance $|\|A - \bar{A}\||$ between an operator A with complex eigenvalues and the closest operator $\bar{A}$ with real eigenvalues is no greater than $\rho \sin(|\theta|/2)$. Hence, if this value is small (compared to 1: it is a quantity without units), then A is "approximately" diagonal in the real field, that is, there is a basis of V such that the action of A on any element u is well approximated by the mere scaling of the coordinates of u in this basis.